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FERMILAB-Pub-97/287-A

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P.O. Box 500, Batavia, Illinois 60510

August 1997

Submitted to *Astrophysical Journal*

Operated by Universities Research Association Inc. under Contract No. DE-AC02-76CH03000 with the United States Department of Energy

Light-cone effect on higher-order clustering in redshift surveys

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ABSTRACT

We have evaluated a systematic effect on counts-in-cells analysis of deep, wide-field galaxy catalogues induced by the evolution of clustering within the survey volume. A multiplicative correction factor was determined, which can be applied after the higher order correlation functions have been extracted in the usual way, without taking into account the evolution. The general theory of this effect combined with the ansatz describing the non-linear evolution of clustering in simulations enables us to estimate the magnitude of the correction factor in different cosmologies. In a series of numerical calculations assuming an array of cold dark matter models, it is found that, as long as galaxies are unbiased tracers of underlying density field, the effect is relatively small ($\simeq 10\%$) for the shallow surveys ($z < 0.2$), while it becomes significant (order of unity) in deep surveys ($z \sim 1$). The required correction is comparable to or smaller than the expected errors of on-going wide-field galaxy surveys such as the SDSS and 2dF. Therefore at present, the effect has to be taken into account for high precision measurements at very small scales only, while in future deep surveys it amounts to a significant correction.

Subject headings: galaxies: clustering - galaxies: distances and redshifts - large-scale structure of Universe - cosmology: theory - dark matter

1. Introduction

Cosmological observations are necessarily carried out on a null hypersurface or a light-cone. At low redshifts ($z < 0.1$), this can be regarded as to provide information on the constant-time hypersurface ($z = 0$) which is a quite conventional implicit approximation underlying cosmological studies using the galaxy redshift surveys. When the depth of the survey volume exceeds $z \sim 0.1$, however, this approximation breaks down, and one should simultaneously consider the intrinsic evolution of galaxy clustering and the light-cone effect in addition to any other selection effect in interpreting the data. This is indeed the case for the on-going wide-field surveys of galaxies including 2dF (2-degree Field Survey) and SDSS (Sloan Digital Sky Survey).

To our knowledge, the first quantitative consideration of the light-cone effect is made by Nakamura, Matsubara, & Suto (1997) who derived the systematic bias in the estimate of $\beta \approx \Omega_0^{0.6}/b$ from magnitude-limited surveys of galaxies combining the cosmological redshift distortion effect (Matsubara & Suto 1996) and the evolution of galaxy clustering within the survey volume. In this paper, we examine the light-cone effect on higher-order statistics of galaxy clustering, considering counts-in-cells analysis specifically.

Since the galaxy two-point correlation function does not contain phase information, the N-point correlation functions are needed for full description. However, they have complex dependence on their arguments, which makes their measurement and interpretation difficult. This is why the normalized, spatially averaged N-point correlation functions, S_N's (see e.g., Peebles 1980) became some of the most successful statistics for describing the higher order properties of the galaxy density field. Next, we introduce the basic formulae needed to investigate the light-cone effect on these objects.

Let us consider first the higher-order statistics on the idealistic constant-time hypersurface. Denote the N-th order correlation functions at a redshift z by $\xi_N(r_1, \ldots, r_N; z)$. Its volume-average is written as

$$\overline{\xi}_N(R; z) = \frac{1}{v^N} \int_v d^3r_1 \cdots d^3r_N \xi_N(r_1, \ldots, r_N; z), \tag{1}$$

where R is the comoving smoothing length to characterize the cell volume v. It is conventional to use the normalized higher-order moments:

$$S_N(R; z) \equiv \frac{\overline{\xi}_N(R; z)}{\left[\overline{\xi}_2(R; z)\right]^{N-1}}. \tag{2}$$

The hierarchical clustering ansatz states that $S_N(R; z)$ is constant and independent of the scale R. This is a good approximation in nonlinear regimes, although a small but definite scale-dependence is clearly detected from N-body experiments (Lahav et al. 1993; Suto 1993; Matsubara & Suto 1994; Suto & Matsubara 1994; Jing & Börner 1997). In addition, perturbation theory predicts that $\overline{\xi}_N(R; z)$ evolves in proportion to $\left[\overline{\xi}_2(R; z)\right]^{N-1}$, and therefore $S_N(R; z)$ is independent of time, i.e. it is constant with respect to z.

The next section exposes the general theory of the light cone effect on $S_N(R;z)$ defined above. Using the ansatz by Jain, Mo, & White (1995; hereafter JMW), §3 evaluates the appropriate correction in an array of cold dark matter (CDM) models. Finally, §4 summarizes the results and discusses the implications for redshift surveys.

2. Observing the higher-order moments on the light-cone

It is difficult to estimate $\overline{\xi}_N(R;z)$ or $\xi_N(r_1,\ldots,r_N;z)$ observationally since z is changing over the volume of galaxy sample. While in principle one could measure the N-point functions on $z = const$ surfaces, in practice this would result in a diminished volume, thus a significant increase of the errors. Instead it is more practical to extract the following N-th order correlation functions averaged over the volumes on the light-cone:

$$\overline{\xi}_N(R;<z_{\rm max}) \equiv \frac{\displaystyle\int_0^{z_{\rm max}} z^2 dz\ w(z)\,\overline{\xi}_N(R;z)}{\displaystyle\int_0^{z_{\rm max}} z^2 dz\ w(z)}. \tag{3}$$

In the above expression, we assume that the observation is performed with the fixed solid angle, and the sampling cells for the analysis are placed randomly in z-coordinate with $w(z)$ being its weighting function. If the cells were located randomly in the comoving coordinates, the volume element z^2dz should have been replaced by $d_A(z;\Omega_0,\lambda_0)^2 c|dt/dz|dz$ (d_A is the angular diameter distance; see, Nakamura et al. 1997) and thus the procedure itself becomes dependent on adopted values of Ω_0 and λ_0.

In principle $w(z)$ is an arbitrary function, and should be determined so as to maximize the signal-to-noise ratio given the selection function of individual observation. By $z_{\rm max}$ we denote the redshift corresponding to the depth of the survey. For a volume-limited sample, for instance, it is natural to set $w(z)=1$ and $z_{\rm max}$ as the maximum redshift of the sample.

Similarly we define the (observable) higher-order moments averaged over the light-cone as

$$\overline{S_N}(R;<z_{\rm max}) \equiv \frac{\overline{\xi}_N(R;<z_{\rm max})}{\left[\overline{\xi}_2(R;<z_{\rm max})\right]^{N-1}}. \tag{4}$$

It is useful to introduce the function $G(z)$ which describes the evolution of the averaged two-point correlation function:

$$\overline{\xi}(R;z) = G(z)\overline{\xi}(R;0). \tag{5}$$

In linear regime, $G(z)$ is equivalent to $[D(z)/D(0)]^2$ where $D(z) = D(z;\Omega_0,\lambda_0)$ is the linear growth rate:

$$\begin{aligned} D(z;\Omega_0,\lambda_0) \quad &\equiv \quad \sqrt{\Omega_0(1+z)^3 + (1-\Omega_0-\lambda_0)(1+z)^2+\lambda_0} \\ &\qquad \times \int_z^\infty \frac{(1+z')\ dz'}{[\Omega_0(1+z')^3+(1-\Omega_0-\lambda_0)(1+z')^2+\lambda_0]^{3/2}}. \end{aligned} \tag{6}$$

Although the above relation (5) does not exactly hold in the nonlinear regime, several approximation formulae are derived in the literature, which empirically describe the evolution by allowing $G(z)$ depend on the scale R (see §3 for details).

If we adopt the evolution law (5), equation (4) is written as

$$\overline{S_N}(R;<z_{\rm max}) = \frac{\displaystyle\int_0^{z_{\rm max}} z^2 dz\; w(z) S_N(R;z)\,\{G(z)\}^{N-1} \left[\int_0^{z_{\rm max}} z^2 dz\; w(z)\right]^{N-2}}{\displaystyle\left[\int_0^{z_{\rm max}} z^2 dz\; w(z) G(z)\right]^{N-1}}. \tag{7}$$

If $z_{\rm max} \ll 1$, the above expression is expanded in terms of $z_{\rm max}$ as follows:

$$\begin{aligned}\overline{S_N}(R;<z_{\rm max}) &= S_N(R;0) + \frac{3}{4} S'_N(0) z_{\rm max} \\ &+ \left[\frac{3}{160}(N-1)(N-2)S_N(0)G'(0)^2 + \frac{3}{80}(N-1)S'_N(0)G'(0) + \frac{3}{10}S''_N(0)\right] z_{\rm max}^2 \\ &+ \mathcal{O}(z_{\rm max}^3),\end{aligned} \tag{8}$$

where $S'_N(0)$ denotes $\partial S_N(R;z)/\partial z|_{z=0}$ and so on. The above expansion up to $\mathcal{O}(z_{\rm max}^2)$ is valid as long as the weighting function is well-approximated up to the same order:

$$w(z_{\rm max}) = w(0) + w'(0) z_{\rm max} + \frac{1}{2} w''(0) z_{\rm max}^2. \tag{9}$$

In other words, $z_{\rm max}$ should be set to be smaller than the effective window size of $w(z_{\rm max})$.

It is interesting to note that up to $\mathcal{O}(z_{\rm max}^2)$ equation (8) is independent of $w(z)$, and that $\mathcal{O}(z_{\rm max})$ term is determined only by $S'_N(0)$ independently of $G(z)$. Since $S'_N(0)$ is expected to vanish in linear theory (Fry 1984; Goroff et al. 1986, Bouchet et al. 1992; Bernardeau 1992), and shown to be relatively small even in nonlinear regimes (Lahav et al. 1993 ; Colombi, Bouchet, & Hernquist 1995, Szapudi et al. 1997), equation (8) implies that the light-cone effect is very small for 2dF and SDSS galaxy redshift surveys ($z_{\rm max} < 0.2$). It should be noted, however, that if galaxies are biased relative to the mass density field, $S'_N(0)$ may not be necessarily small. So any signal proportional to $z_{\rm max}$ provides a clear indication of the time-dependent biasing of galaxies (see e.g., Fry 1996; Mo & White 1996; Mo, Jing & White 1997) independent of $G(z)$ and $w(z)$.

3. Evaluating the light-cone effect: an example

In order to evaluate the effect of observational average on the light-cone, we assume that $S_N(R;z)$ does not evolve with z, i.e., $S_N(R;z) = S_N(R;0)$. As described above, this is a reasonable approximation as long as galaxies are unbiased tracers of underlying density field. If we introduce the measure of the light-cone effect:

$$\Delta_N(R;<z_{\rm max}) \equiv \frac{\overline{S_N}(R;<z_{\rm max})}{S_N(R;0)} - 1, \tag{10}$$

equations (7) and (8) with $S_N(R;z) = S_N(R;0)$ reduce to

$$\Delta_N(z_{\rm max}) = \frac{3}{160}(N-1)(N-2)G'(0)^2 z_{\rm max}^2 + \mathcal{O}(z_{\rm max}^3). \tag{11}$$

Note that $(1+\Delta_N)$ can be regarded as a correction factor as well, if one measures the S_N's *without* considering the evolution of clustering. This constitutes a simple and practical method, which we propose for future measurements, when compensation for the light cone effect is needed.

To evaluate the equation (7), we need a model for the evolution of variance $G(z)$. For this purpose, we adopt the ansatz originally put forward by Hamilton et al. (1991) and improved later by Peacock & Dodds (1994), JMW and Peacock & Dodds (1996). To be specific, we apply the fitting formula by JMW which relates the evolved two-point correlation function $\overline{\xi}_E(R;z)$ with its linear counterpart $\overline{\xi}_L(R_0;z)$ as follows:

$$\begin{aligned} \overline{\xi}_E(R;z) &= B(n)\,F[\overline{\xi}_L(R_0;z)/B(n)], & (12)\\ R_0 &= \left[1+\overline{\xi}_E(z,R)\right]^{1/3} R, & (13)\\ B(n) &= \left(\frac{3+n}{3}\right)^{0.8}, & (14)\\ F(x) &= \frac{x+0.45x^2-0.02x^5+0.05x^6}{1+0.02x^3+0.003x^{9/2}}. & (15) \end{aligned}$$

In the above equations, n denotes a power-law index of the power spectrum. JMW show that the above formula works reasonably well even for CDM models by replacing n by the effective spectral index evaluated at the scale which is just entering nonlinear regime. Generally, the resulting n depends on z, but we neglect that dependence for simplicity. For galaxy surveys which we are primarily interested in, the z-dependence of n near $z=0$ is expected to be very small.

The inverse of equation (12) is formally written as

$$\overline{\xi}_L(R_0;z) = B(n)\,F^{-1}[\overline{\xi}_E(R;z)/B(n)], \tag{16}$$

and JMW also gives an empirical fit to $F^{-1}(y)$

$$F^{-1}(y) = y\left(\frac{1+0.036y^{1.93}+0.0001y^3}{1+1.75y-0.0015y^{3.63}+0.028y^4}\right)^{1/3}. \tag{17}$$

Using equation (16), one obtains

$$\overline{\xi}_{\rm L}(R_0;z) = \frac{D^2(z)}{D^2(0)}\overline{\xi}_{\rm L}(R_0;0) = \frac{D^2(z)}{D^2(0)}B(n)F^{-1}[\overline{\xi}_E(R;0)/B(n)]. \tag{18}$$

Therefore we can express $\overline{\xi}_E(R;z)$ in terms of $\overline{\xi}_E(R;0)$ as

$$\overline{\xi}_E(R;z) = B(n)F\left[\frac{D^2(z)}{D^2(0)}F^{-1}[\overline{\xi}_E(R,0)/B(n)]\right]. \tag{19}$$

Let us introduce a parameter $\alpha(R) \equiv F^{-1}\left[\overline{\xi}_E(R,0)/B(n)\right]$ which characterizes the variance on a scale R at $z=0$, and thus depends on Ω_0 and λ_0 through the shape of the fluctuation spectrum. Then the scale-dependent evolution factor $G(z)=G(R;z)$ in equation (5) is given by

$$G(R;z) \equiv \frac{\overline{\xi}_E(R;z)}{\overline{\xi}_E(R;0)} = \frac{1}{F(\alpha)} F\left[\frac{D^2(z)}{D^2(0)}\alpha\right]. \tag{20}$$

For the convenience of z-expansion, we calculate the derivatives of the above quantity at $z=0$:

$$\left.\frac{\partial G(R;z)}{\partial z}\right|_{z=0} = -2f_0\frac{\alpha F'(\alpha)}{F(\alpha)}, \tag{21}$$

$$\left.\frac{\partial^2 G(R;z)}{\partial z^2}\right|_{z=0} = 4f_0^2\frac{\alpha^2 F''(\alpha)}{F(\alpha)} + \left(2f_0^2 + 2f_0q_0 + 3\Omega_0\right)\frac{\alpha F'(\alpha)}{F(\alpha)}. \tag{22}$$

where

$$f_0 = \left.\frac{d\ln D}{da}\right|_{z=0}, \qquad q_0 = \frac{\Omega_0}{2} - \lambda_0. \tag{23}$$

The above expressions indicate how the light-cone effect depends on Ω_0 and λ_0 at $z_{\rm max} \ll 1$. Note, that they are involved in $\mathcal{O}(z_{\rm max}^2)$ term and thus do not contribute significantly at small z.

Using equations (7) and (20) and assuming $S_N(z) = S_N(0)$, we can evaluate the evolutionary effect on $\overline{S_N}(R;<z_{\rm max})$ or $\Delta_N(<z_{\rm max})$. As examples, we consider three representative CDM models (Table 1) whose fluctuation amplitude σ_8 is normalized so as to reproduce the abundances of clusters of galaxies (Kitayama & Suto 1997; Kitayama, Sasaki & Suto 1997; see also Viana & Liddle 1996; Eke, Cole, & Frenk 1996). The results are displayed on a series of figures.

Figure 1 shows how α is related to the comoving smoothing length R in these models; $\alpha(R)$ is plotted against $\log_{10} R$ for SCDM, OCDM, and LCDM with solid, thin, and think dotted lines, respectively. In the rest of the figures, these models are plotted on the upper, middle and lower panels, respectively, while different orders, $N = 3\ldots 10$, are plotted as a series of monotonically increasing curves, and $N=3$ and $N=7$ emphasized with thick lines. Figure 2 displays $\Delta_N(R;z)$ as a function of $\log_{10} z$ at $R = 1h^{-1}$Mpc (left panels) and at $R = 10h^{-1}$Mpc (right panels). Finally, Figure 3 plots $\Delta_N(R;z)$ against $\log_{10} R$ for $z=0.2$ (left panel), and $z=1$ (right panel). A discussion of the figures follows in the next section.

4. Conclusions

The general appearance of the figures suggests that the light-cone effect is a fairly robust feature, although its details depend on the model. In all cases SCDM appears to give the strongest effect, while for OCDM and LCDM it is slightly less pronounced; however the difference is fairly small, qualitatively all models behave similarly. Note also, that the magnitude of the correction depends on the order N, and, in accord with intuition, it is monotonically increasing for higher order.

As expected, the light-cone effect becomes larger as $z_{\rm max}$ increases, which can be seen in Figure 2. Although the correction is relatively small for shallow surveys with $z \lesssim 0.2$ samples, $\Delta_N(R; < z_{\rm max})$ becomes $\gtrsim 10\%$ in nonlinear scales ($R \sim 1h^{-1}$Mpc). In SCDM, for instance, $\Delta_N(R; < z_{\rm max})$ exceeds unity for $N \geq 6$ for the entire dynamic range plotted. Furthermore Figure 3 indicates that even if the hierarchical ansatz is correct, i.e., $S_N(R; z)$ is independent of R, the light-cone effect should generate apparent scale-dependence, since the correction behaves differently at different scales at a given redshift.

The future SDSS will be able to measure the moments of the galaxy density field with unprecedented accuracy. If unforeseen systematics does not prevent it, it will determine them with less than a few percent error for $N \leq 3$ and 10% for $N = 4$ between $1h^{-1}$Mpc$\ldots 50h^{-1}$Mpc (see Colombi, Szapudi, & Szalay 1997 for details). According to Figures 2, and 3, the light cone effect will be much smaller than these errors, or at most same order, depending on the scale and model. The correction could be potentially non-negligible only at the smallest scales. Similar conclusion can probably be drawn about the 2dF survey. On the other hands, for future deep surveys, which probably will be aimed at smaller scales, especially if carried out by Space Telescope or like instruments, our calculations will be of utmost importance. According to Figure 3 the required correction can range from up to unity for S_3 through factors of few for S_6 to factors of hundred for S_{10}.

This research was supported in part by the Grants-in-Aid for the Center-of-Excellence (COE) Research of the Ministry of Education, Science, Sports and Culture of Japan (07CE2002) to RESCEU (Research Center for the Early Universe), University of Tokyo. I.S. was supported by DOE and NASA through grant NAG-5-2788 at Fermilab.

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Table 1: CDM model parameters

Model	Ω_0	λ_0	h	σ_8
SCDM	1.0	0.0	0.5	0.6
OCDM	0.45	0.0	0.7	0.8
LCDM	0.3	0.7	0.7	1.0

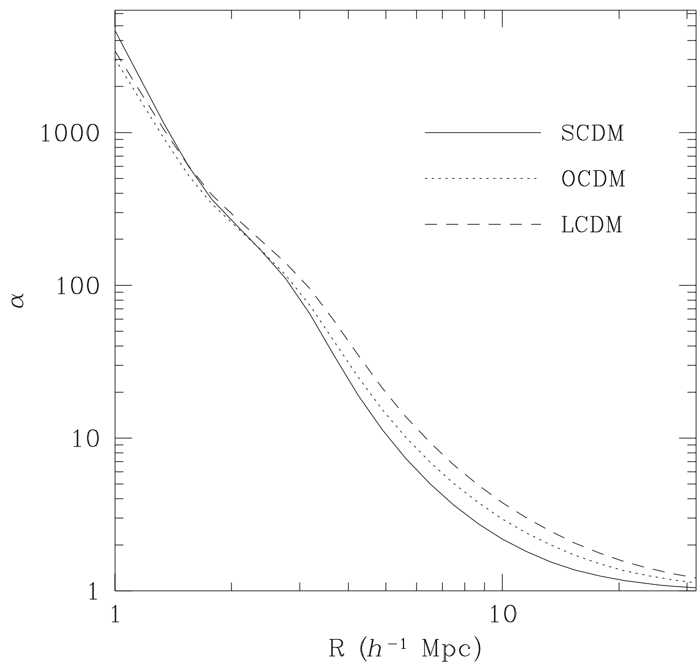

Fig. 1.— $\alpha(R)$ is plotted against $\log_{10} R(1h^{-1}\mathrm{Mpc})$ for SCDM (solid line), OCDM (thin dotted line), and LCDM (thick dotted line) summarized in Table 1.

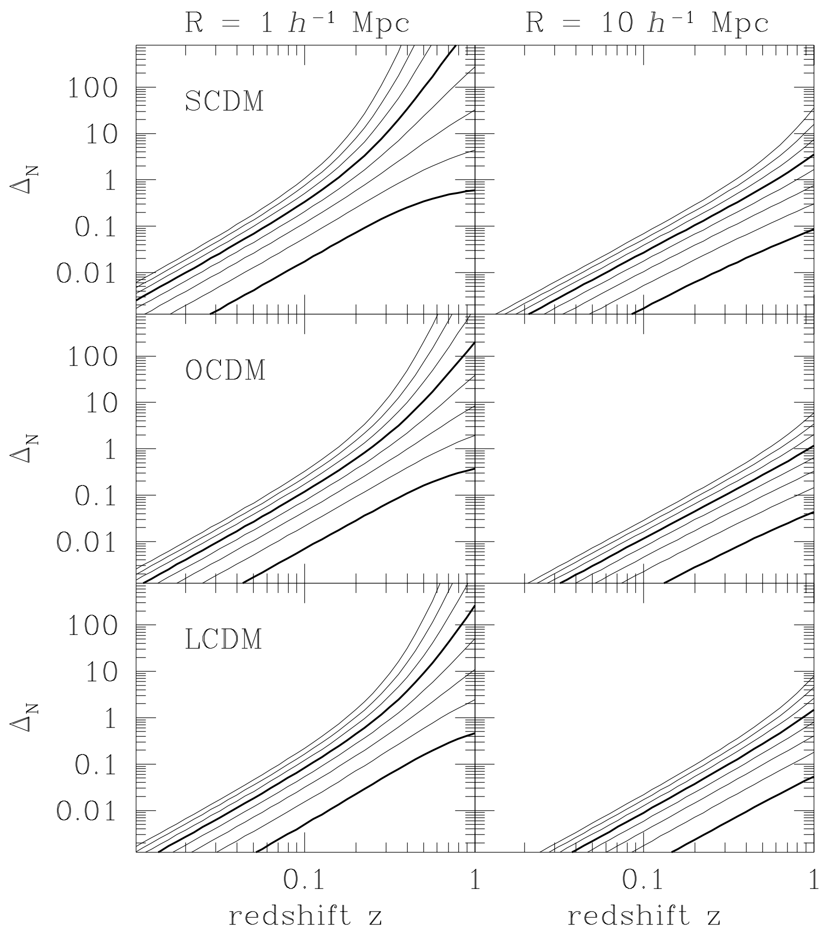

Fig. 2.— $\log_{10} \Delta_N(R; z)$ are shown as functions of $\log_{10} z$ at $R = 1h^{-1}$Mpc (*left panels*) and $10h^{-1}$Mpc (*right panels*); SCDM (*top panels*), OCDM (*middle panels*), and LCDM (*bottom panels*). The family of curves display different orders from $N = 3 \ldots N = 10$ monotonically upward; for $N = 3$, and $N = 7$ is plotted with thick lines for orientation.

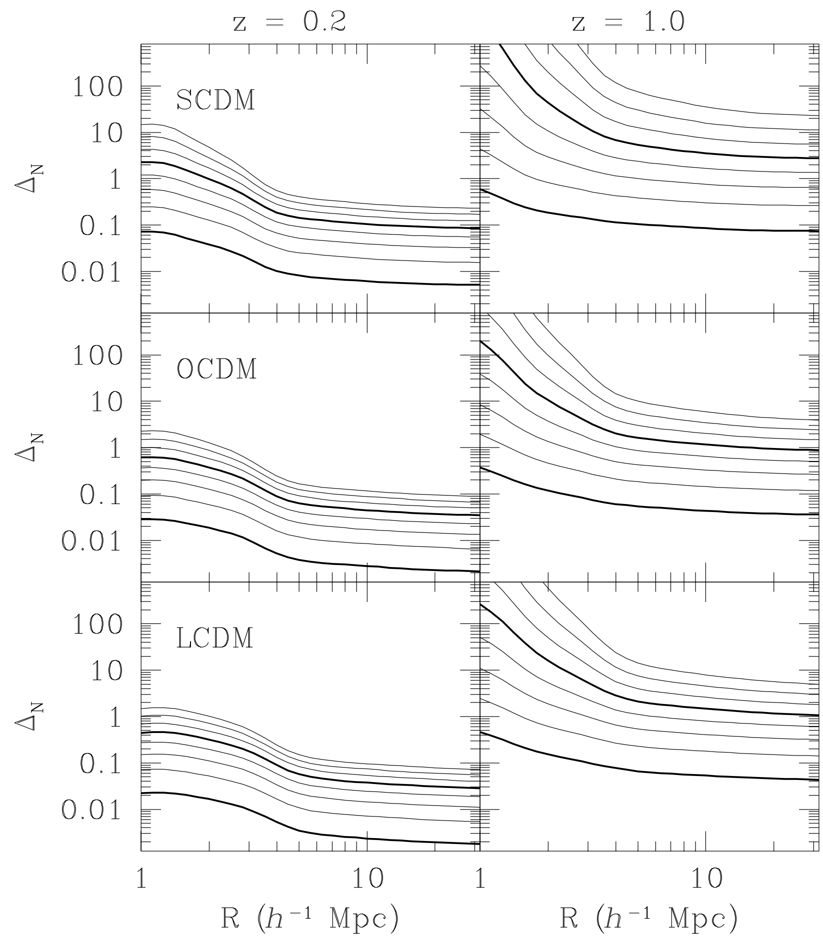

Fig. 3.— $\log_{10} \Delta_N(R; z)$ are displayed as functions of $\log_{10} R$ at $z = 0.2$ (*left panels*) and 1.0 (*right panels*); SCDM (*top panels*), OCDM (*middle panels*), and LCDM (*bottom panels*). The family of curves is the same as for Fig.2.